Complex Numbers

What ARE they?

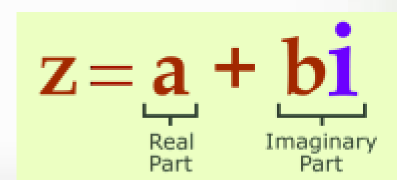
What is a complex number?

- We know that √4 = 2, √25 = 5, √100 = 10

But what is the √-4? How can a number multiplied by itself give a negative number? <u>That's impossible!</u>

So…… √-4 is not a real number.

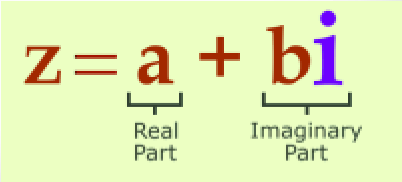
Numbers such as √-1, √-4 and √-25 are called <u>imaginary numbers</u>



Imaginary numbers

- $i = \sqrt{-1}$ and $i^2 = -1$

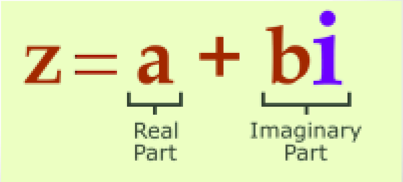
- Now we can write imaginary numbers e.g. √9 in a new form using the symbol i for √-1
- $\sqrt{-9} = \sqrt{9} \, . \, \sqrt{-1} = 3i$
- $\sqrt{-25} = \sqrt{5} \, . \, \sqrt{-1} = 5i$
- $\sqrt{-18} = \sqrt{18} \, . \, \sqrt{-1} = \sqrt{9} \, . \, \sqrt{2} \, . \, i = 3\sqrt{2}i$



Example of a complex number

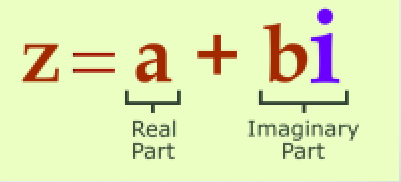
- A number like *3 + 4i* is called a complex number.
- 3 is called the real part.
- 4 is called the imaginary part.

- A complex number is a number of the form $a + bi$, where $i = \sqrt{-1}$



Complex numbers cont.

- Take note! The real number 4 may be written in the form $4 + 0i$ (the real part = 4 and the imaginary part = 0)
- The imaginary number $-3i$ may be written in the form $0 - 3i$ (the real part = 0 and the imaginary part = -3)
- The capital letter C is used to represent the set of complex numbers.
- The letter z is generally used to represent a complex number, e.g. $z_1 = 3 - 4i$, $z_2 = -5 + 6i$



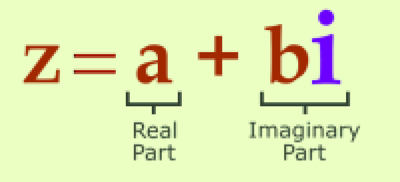
Adding and subtracting complex numbers

- To add or subtract two complex numbers, add or subtract the real parts and the imaginary parts separately, for example:

$(3 + 2i) + (4 – 3i)$

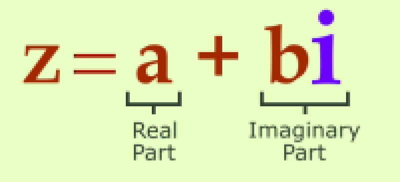
$= 3 + 2i + 4 – 3i$ *(add the real numbers and then add the imaginary numbers)*

$= 7 - i$



Removing the brackets

- We can simplify ***5(3 – 4i)*** by multiplying each term in the brackets by 5.
- = ***15 – 20i***



An example

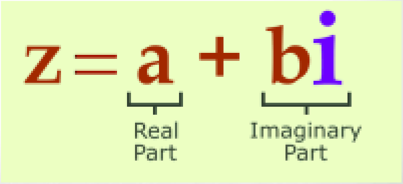
- If $z_1 = 1 + 2i$ *and* $z_2 = 5 - 2i$, express in the terms a + bi:

$z_1 + z_2$

$= (1 + 2i) + (5 - 2i)$

$= 1 + 2i + 5 - 2i$

$= 6 + 0i$



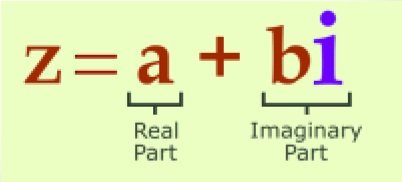
Multiplying complex numbers

- We multiply complex numbers in the same way we multiply algebraic expressions, but we replace the i^2 with -1.
- Example: $i(4 + 2i)$

$$= 4i + 2i^2$$

$$= 4i + 2(-1)$$

$$= 4i - 2$$



Example of multiplying complex numbers

- $(4 + i)(5 - 2i)$

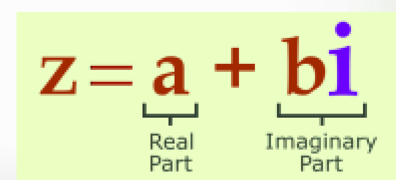
$= 4(5 - 2i) + i(5 - 2i)$

$= 20 - 8i + 5i - 2i^2$

$= 20 + 3i - 2(-1)$ *(Replace i^2 with -1)*

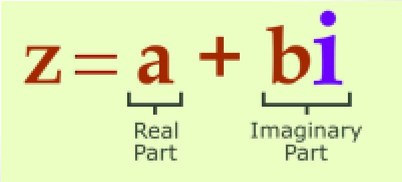
$= 20 + 3i + 2$

$= 22 + 3i$



Dividing complex numbers

- If $z = a + bi$ is a complex number, then $a - bi$ is called the complex conjugate of z, or simply the conjugate of z.
- It sounds complicated, but just change the sign of the imaginary number to get the conjugate!
- Examples: The conjugate of $3 - 4i$ is $3+4i$ and the conjugate of $-2 + 5i$ is $-2 - 5i$. That's simple!
- We write the conjugate of z like this: $\bar{z}$



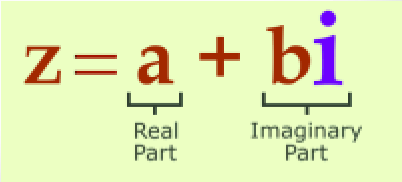
Dividing complex numbers cont.

- Express in the form a + bi:

$$\frac{4 - 3i}{3 + 2i}$$

- To express this in this form, we multiply above and below by the conjugate bottom number.

$$\frac{4 - 3i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$$



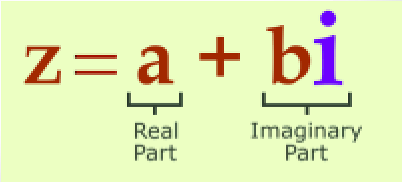
Example cont.

$$= \frac{(4 - 3i)(3 - 2i)}{(3 + 2i)(3 - 2i)}$$

$$= \frac{4(3 - 2i) - 3i(3 - 2i)}{3(3 - 2i) + 2i(3 - 2i)}$$

$$= \frac{12 - 8i - 9i + 6(-1)}{9 - 6i + 6i - 4(-1)}$$

$$= \frac{6 + 17i}{13} = \frac{6}{13} + \frac{17}{13} i$$



Simples!

$$z = a + bi$$

Real Part (a) — Imaginary Part (bi)